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On Black Holes and Cosmological Constant in Noncommutative Gauge Theory of Gravity*

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Abstract: Deformed Reissner-Nordström, as well as Reissner-Nordström de Sitter, solutions are obtained in a noncommutative gauge theory of gravitation. The gauge potentials (tetrad fields) and the components of deformed metric are calculated to second order in the noncommutativity parameter. The solutions reduce to the deformed Schwarzschild ones when the electric charge of the gravitational source and the cosmological constant vanish. Corrections to the thermodynamical quantities of the corresponding black holes and to the radii of different horizons have been determined. All the independent invariants, such as the Ricci scalar and the so-called Kretschmann scalar, have the same singularity structure as the ones of the usual undeformed case and no smearing of singularities occurs. The possibility of such a smearing is discussed. In the noncommutative case we have a local disturbance of the geometry around the source, although asymptotically at large distances it becomes flat.

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Contents

1. Introduction

Questioning the nature of space-time at infinitely small scales has been a fundamental issue for physics. It is generally believed that the visionary Riemann hinted to a possible breakdown of space-time as a manifold already in 1854, in his famous inaugural lecture [1]. The quantum nature of space-time, expressed as noncommutativity of space-time coordinates, has been lately a subject of active research, especially in connection with string theory [2].

Naturally, various effects of space-time noncommutativity in cosmology have been studied, principally motivated by the fact that, since noncommutativity is believed to be significant at the Planck scale - the same scale where quantum gravity effects become important - it is most sensible to search for signatures of noncommutativity in the cosmological observations (for a review, see [3] and references therein). One of the most compelling reasons for the study of noncommutative inflation is the fact that in an inflationary model the physical wavelengths observed today in cosmological experiments emerged from the Planckian region in the early stages of inflation, and thus carry the effects of the Planck scale physics [4]. Among other things, the observed anisotropies of the cosmic microwave background (CMB) may be caused by the noncommutativity of space-time [5, 6]. The noncommutativity has been taken into account either through space-space uncertainty relations [7] or space-time uncertainty relations [8], as well as noncommutative description of the inflaton (with gravity as background which is not affected by noncommutativity) [5]. On the side of noncommutative black-hole physics, the studied effect of noncommutativity was the smearing of the mass-density of a static, spherically-symmetric, particle-like gravitational source [9] (see also [10]).

These very interesting ideas have been developed lacking a noncommutative theory of gravity. Although various proposals have been made (see, for a list of references, [11]), an ultimate noncommutative theory of gravity is still elusive. We believe that the most natural way towards this goal is the gauging of the twisted Poincaré symmetry [12]. Although the formulation of twisted internal gauge theories has not yet been achieved [13], the possibility of gauging the (space-time) twisted Poincaré algebra has not been ruled out and the issue is under investigation.

At the moment, one of the most coherent approaches to noncommutative gravity is the one proposed by Chamseddine [14], consisting in gauging the noncommutative $SO(4,1)$ de Sitter group and using the Seiberg-Witten map with subsequent contraction to the Poincaré (inhomogeneous Lorentz) group $ISO(3,1)$. Although this formulation is not a final theory of noncommutative gravity, it still can serve as a concrete model to be studied, whose main features shall illustrate at least qualitatively the influence of quantum space-time on gravitational effects. The study of specific examples as such can cast light upon the reasonable and unreasonable assumptions proposed so far in the field. Besides, up to now, there have been no calculations presented in the literature (except [11]) to obtain the metric by solving a NC version of gravitational theory, dues to the technical difficulty of the task.

In a recent paper [11] a deformed Schwarzschild solution in noncommutative gauge theory of gravitation was obtained based on [14]. The gravitational gauge potentials (tetrad fields) were calculated for the Schwarzschild solution and the corresponding deformed metric $\hat{g}_{\mu\nu}(x,\Theta)$ was defined. According to the result of [11] corrections appear only in the second order of the expansion in Θ, i.e. there are no first order correction terms.

In this paper we attempt to extend the results of [11] to include as well the Reissner-Nordström solution. Having these two classical solutions known in the noncommutative setup, we can embark upon a more rigorous study of noncommutative black-hole physics. Black hole thermodynamical quantities depend on the Hawking temperature via the usual thermodynamical relations. The Hawking temperature undergoes corrections from many sources: the quantum corrections [15], the self-gravitational corrections [16], and the corrections due to the generalized uncertainty principle [17]. In this paper we focus on the corrections due to the space-space noncommutativity.

The results of this paper have been obtained using a program devised for GRTensor II application of Maple. For the self-consistence of the paper we shall present, in the commutative case, results of the de Sitter gauge theory with spherical symmetry, obtained with a similar type of program [18] and recall the derivation of the metric tensor components in the noncommutative case, as obtained in [11].

2. de Sitter gauge theory with spherical symmetry

2.1 Commutative case

In the following we shall sketch the principal aspects of a model of gauge theory for gravitation having the de Sitter group (dS) as local symmetry and gravitational field created by a point-like source of mass m and carrying also the electric charge Q. The detailed

treatment, including the analytical GRTensor II program used for the calculations, can be found in Ref. [18].

The base manifold is a four-dimensional Minkowski space-time M_4, in spherical coordinates:

$$ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{2.1}$$

The corresponding metric $g_{\mu\nu}$ has the following non-zero components:

$$g_{00} = -1, \qquad g_{11} = 1, \qquad g_{22} = r^2, \qquad g_{33} = r^2\sin^2\theta. \tag{2.2}$$

The infinitesimal generators of the 10-dimensional de Sitter group will be denoted by Π_a and $M_{ab} = -M_{ba}$, $a, b = 1, 2, 3, 0$, where Π_a generate the de Sitter "translations" and M_{ab} - the Lorentz transformations. In order to give a general formulation of the gauge theory for the de Sitter group dS, we will denote the generators Π_a and M_{ab} by X_A, $A = 1, 2, \ldots, 10$. The corresponding 10 gravitational gauge fields will be the tetrads $e^a_\mu(x)$, $a = 0, 1, 2, 3$, and the spin connections $\omega^{ab}_\mu(x) = -\omega^{ba}_\mu(x)$, $[ab] = [01], [02], [03], [12], [13], [23]$. Then, the corresponding components of the strength tensor can be written in the standard form, as the torsion tensor:

$$F^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu + \left(\omega^{ab}_\mu e^c_\nu - \omega^{ab}_\nu e^c_\mu\right)\eta_{bc}\,, \tag{2.3}$$

with η_{ab} the flat space metric, and the curvature tensor:

$$F^{ab}_{\mu\nu} \equiv R^{ab}_{\mu\nu} = \partial_\mu\omega^{ab}_\nu - \partial_\nu\omega^{ab}_\mu + \left(\omega^{ac}_\mu\omega^{db}_\nu - \omega^{ac}_\nu\omega^{db}_\mu\right)\eta_{cd} + 4\lambda^2\left(\delta^b_c\delta^a_d - \delta^a_c\delta^b_d\right)e^c_\mu e^d_\nu, \tag{2.4}$$

where λ is a real parameter. The integral of action associated to the gravitational gauge fields $e^a_\mu(x)$ and $\omega^{ab}_\mu(x)$ will be chosen as [19]:

$$S_g = \frac{1}{16\pi G}\int d^4x\, e\, F, \tag{2.5}$$

where $e = \det(e^a_\mu)$ and

$$F = F^{ab}_{\mu\nu}\, e^\mu_a\, e^\nu_b. \tag{2.6}$$

Here, $e^\mu_a(x)$ denotes the inverse of $e^a_\mu(x)$ satisfying the usual properties:

$$e^a_\mu e^\mu_b = \delta^a_b, \qquad e^a_\mu e^\nu_a = \delta^\nu_\mu. \tag{2.7}$$

We assume that the source of the gravitation creates also an electromagnetic field $A_\mu(x)$, with the standard action [20]:

$$S_{em} = -\frac{1}{4Kg^2}\int d^4x\, e\, A^a_\mu\, A^\mu_a, \tag{2.8}$$

where $A^a_\mu = A^\nu_\mu e^a_\nu$, $A^\nu_\mu = e^\nu_a e^\rho_b \eta^{ab} A_{\mu\rho}$ and respectively $A^\mu_a = A^\nu_\mu e^\nu_a$, with $A_{\mu\rho}$ being the electromagnetic field tensor, $A_{\mu\rho} = \partial_\mu A_\rho - \partial_\rho A_\mu$. Here K is a constant that will be chosen in a convenient form to simplify the solutions of the field equations and g is the gauge coupling constant [20].

Then, the total integral of action associated to the system composed of the two fields is given by the sum of the expressions (2.5) and (2.8):

$$S = \int d^4x \left[\frac{1}{16\pi G} F - \frac{1}{4Kg^2} A_\mu^a \, A_a^\mu \right] e. \tag{2.9}$$

The field equations for the gravitational potentials $e_\mu^a(x)$ are obtained by imposing the variational principle $\delta_e S = 0$ with respect to $e_\mu^a(x)$. They are [21]:

$$F_\mu^a - \frac{1}{2} F \, e_\mu^a = 8\pi G T_\mu^a, \tag{2.10}$$

where F_μ^a is defined by:

$$F_\mu^a = F_{\mu\nu}^{ab} \, e_b^\nu, \tag{2.11}$$

and T_μ^a is the energy-momentum tensor of the electromagnetic field [22]:

$$T_\mu^a = \frac{1}{Kg^2} \left(A_\mu^b \, A_\nu^a \, e_b^\nu - \frac{1}{4} A_\nu^b \, A_b^\nu \, e_\mu^a \right). \tag{2.12}$$

The field equations for the other gravitational gauge potentials $\omega_\mu^{ab}(x)$ are equivalent with:

$$F_{\mu\nu}^a = 0. \tag{2.13}$$

The solutions of the field equations (2.10) and (2.13) were obtained in [18], under the assumption that the gravitational field has spherical symmetry and it is created by a point-like source of mass m, which also produces, due to its constant electric charge Q, the electromagnetic field $A_\mu(x)$. The particular form of the spherically symmetric gravitational gauge field adopted in [18] is given by the following Ansatz:

$$e_\mu^0 = (A, 0, 0, 0), \quad e_\mu^1 = \left(0, \frac{1}{A}, 0, 0 \right), \quad e_\mu^2 = (0, 0, r, 0), \quad e_\mu^3 = (0, 0, 0, r \sin\theta), \tag{2.14}$$

and

$$\begin{gathered} \omega_\mu^{01} = (U, 0, 0, 0), \quad \omega_\mu^{02} = \omega_\mu^{03} = 0, \quad \omega_\mu^{12} = (0, 0, A, 0), \\ \omega_\mu^{13} = (0, 0, 0, A \sin\theta), \quad \omega_\mu^{23} = (0, 0, 0, \cos\theta), \end{gathered} \tag{2.15}$$

where A and U are functions only of the 3D radius r. With the above expressions the components of the tensors $F_{\mu\nu}^a$ and $F_{\mu\nu}^{ab}$ defined by the Eqs. (2.3) and (2.4) were computed. Here we give only the expressions of $F_{\mu\nu}^{ab}$ components, which we need to use further, in the derivation of the expressions of the deformed tetrads:

$$\begin{aligned} F_{10}^{01} &= U' + 4\lambda^2, \qquad F_{20}^{02} = A \left(U + 4\lambda^2 r \right), \qquad F_{30}^{03} = A \sin\theta \left(U + 4\lambda^2 r \right), \\ F_{21}^{12} &= \frac{-AA' + 4\lambda^2 r}{A}, \qquad F_{31}^{13} = \frac{\left(-AA' + 4\lambda^2 r \right) \sin\theta}{A}, \\ F_{32}^{23} &= \left(1 - A^2 + 4\lambda^2 r^2 \right) \sin\theta, \end{aligned} \tag{2.16}$$

where A' and U' denote the derivatives with respect to the variable r.

Using the field equations, the solution is obtained [18] as:

$$\begin{aligned} U &= -AA', \\ A^2 &= 1 + \frac{\alpha}{r} + \frac{Q^2}{r^2} + \beta r^2, \end{aligned} \tag{2.17}$$

where α and β are constants of integration. It is well-known [22] that the constant α is determined by the mass m of the point-like source that creates the gravitational field, by comparison with the Newtonian limit at very large distances:

$$\alpha = -2m. \tag{2.18}$$

The other constant β was determined in [18] (see also [23]) as $\beta = 4\lambda^2 = -\frac{\Lambda}{3}$, where Λ is the cosmological constant, such that the solution finally reads:

$$A^2 = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2, \qquad U = -\frac{m}{r^2} + \frac{Q^2}{r^3} + \frac{\Lambda}{3}r. \tag{2.19}$$

If we consider the contraction $\Lambda \to 0$, then the de Sitter group becomes the Poincaré group, and the solution (2.19) reduces to the Reissner-Nordström one.

2.2 Noncommutative case using the Seiberg-Witten map

The noncommutative corrections to the metric of a space-time with spherically symmetric gravitational field have been obtained in [11], based on the general outline developed by Chamseddine [14].

The noncommutative structure of the space-time is determined by the commutation relation

$$[x^\mu, x^\nu] = i\,\Theta^{\mu\nu}, \tag{2.20}$$

where $\Theta^{\mu\nu} = -\,\Theta^{\nu\,\mu}$ are constant parameters. It is well known that noncommutative field theory on such a space-time requires is defined by introducing the star product "*" between the functions f and g defined over this space-time:

$$(f{*}g)\,(x) = f\,(x)\; e^{\frac{i}{2}\,\Theta^{\mu\nu}\overleftarrow{\partial_\mu}\;\overrightarrow{\partial_\nu}} g\,(x)\,. \tag{2.21}$$

The gauge fields corresponding to the de Sitter gauge symmetry for the noncommutative case are denoted by $\hat{e}^a_\mu\,(x,\,\Theta)$ and $\hat{\omega}^{ab}_\mu\,(x,\,\Theta)$, generically denoted by $\hat{\omega}^{AB}_\mu\,(x,\,\Theta)$, with the obvious meaning for the indices A, B. The main idea of the Seiberg-Witten map is to expand the noncommutative gauge fields, transforming according to the noncommutative gauge algebra, in terms of commutative gauge fields, transforming under the corresponding commutative gauge algebra, in such a way that the noncommutative and commutative gauge transformations are compatible, i.e.

$$\hat{\omega}^{AB}_\mu(\omega) + \delta_{\hat{\lambda}}\hat{\omega}^{AB}_\mu(\omega) = \hat{\omega}^{AB}_\mu(\omega + \delta_\lambda\omega). \tag{2.22}$$

where $\delta_{\hat{\lambda}}$ are the infinitesimal variations under the noncommutative gauge transformations and δ_λ are the infinitesimal variations under the commutative gauge transformations.

Using the Seiberg-Witten map [2], one obtains the following noncommutative corrections up to the second order [14]:

$$\omega_{\mu\nu\rho}^{AB}(x) = \frac{1}{4}\,\{\omega_\nu,\,\partial_\rho\omega_\mu + F_{\rho\mu}\}^{AB}\,, \tag{2.23}$$

$$\begin{aligned}\omega_{\mu\nu\rho\lambda\tau}^{AB}(x) = \frac{1}{32}\,&\big(-\{\omega_\lambda,\partial_\tau\{\omega_\nu,\partial_\rho\omega_\mu + F_{\rho\mu}\}\} + 2\{\omega_\lambda,\{F_{\tau\nu},F_{\mu\rho}\}\}\\ &-\{\omega_\lambda,\{\omega_\nu,D_\rho F_{\tau\mu}+\partial_\rho F_{\tau\mu}\}\} - \{\{\omega_\nu,\partial_\rho\omega_\lambda+F_{\rho\lambda}\},(\partial_\tau\omega_\mu+F_{\tau\mu})\}\\ &+2\,[\partial_\nu\omega_\lambda,\partial_\rho(\partial_\tau\omega_\mu+F_{\tau\mu})]\,\big)^{AB}\,,\end{aligned} \tag{2.24}$$

where

$$\{\alpha,\beta\}^{AB} = \alpha^{AC}\,\beta_C^B + \beta^{AC}\,\alpha_C^B, \quad [\alpha,\beta]^{AB} = \alpha^{AC}\,\beta_C^B - \beta^{AC}\,\alpha_C^B \tag{2.25}$$

and

$$D_\mu F_{\rho\sigma}^{AB} = \partial_\mu F_{\rho\sigma}^{AB} + \left(\omega_\mu^{AC}\,F_{\rho\sigma}^{D\,B} + \omega_\mu^{BC}\,F_{\rho\sigma}^{D\,A}\right)\eta_{CD}. \tag{2.26}$$

The noncommutative tetrad fields were obtained in [14] up to the second order in Θ in the limit $\Lambda \to 0$ as:

$$\hat{e}_\mu^a(x,\Theta) = e_\mu^a(x) - i\,\Theta^{\nu\rho}\,e_{\mu\nu\rho}^a(x) + \Theta^{\nu\rho}\,\Theta^{\lambda\tau}\,e_{\mu\nu\rho\lambda\tau}^a(x) + O\left(\Theta^3\right), \tag{2.27}$$

where

$$e_{\mu\nu\rho}^a = \frac{1}{4}\left[\omega_\nu^{a\,c}\partial_\rho e_\mu^d + \left(\partial_\rho\omega_\mu^{a\,c} + F_{\rho\mu}^{a\,c}\right)e_\nu^d\right]\eta_{c\,d}, \tag{2.28}$$

$$\begin{aligned}e_{\mu\nu\rho\lambda\tau}^a = \frac{1}{32}\Big[&2\{F_{\tau\nu},F_{\mu\rho}\}^{a\,b}\,e_\lambda^c - \omega_\lambda^{a\,b}\left(D_\rho\,F_{\tau\mu}^{c\,d} + \partial_\rho\,F_{\tau\mu}^{c\,d}\right)e_\nu^m\,\eta_{d\,m}\\ &-\{\omega_\nu,(D_\rho F_{\tau\mu}+\partial_\rho F_{\tau\mu})\}^{a\,b}\;e_\lambda^c - \partial_\tau\{\omega_\nu,(\partial_\rho\,\omega_\mu + F_{\rho\mu})\}^{a\,b}\,e_\lambda^c\\ &-\omega_\lambda^{a\,b}\,\partial_\tau\left(\omega_\nu^{c\,d}\,\partial_\rho e_\mu^m + \left(\partial_\rho\,\omega_\mu^{c\,d} + F_{\rho\mu}^{c\,d}\right)e_\nu^m\right)\eta_{dm} + 2\,\partial_\nu\omega_\lambda^{a\,b}\partial_\rho\partial_\tau\,e_\mu^c\\ &-2\,\partial_\rho\left(\partial_\tau\,\omega_\mu^{a\,b} + F_{\tau\mu}^{a\,b}\right)\partial_\nu\,e_\lambda^c - \{\omega_\nu,(\partial_\rho\omega_\lambda + F_{\rho\lambda})\}^{a\,b}\,\partial_\tau\,e_\mu^c\\ &-\left(\partial_\tau\,\omega_\mu^{a\,b} + F_{\tau\mu}^{a\,b}\right)\left(\omega_\nu^{c\,d}\partial_\rho e_\lambda^m + \left(\partial_\rho\,\omega_\lambda^{c\,d} + F_{\rho\lambda}^{c\,d}\right)e_\nu^m\,\eta_{d\,m}\right)\Big]\,\eta_{b\,c}.\end{aligned} \tag{2.29}$$

Using the hermitian conjugate $\hat{e}_\mu^{a\dagger}(x,\Theta)$ of the deformed tetrad fields given in (2.27),

$$\hat{e}_\mu^{a\,\dagger}(x,\Theta) = e_\mu^a(x) + i\,\Theta^{\nu\rho}\,e_{\mu\nu\rho}^a(x) + \Theta^{\nu\rho}\Theta^{\lambda\tau}e_{\mu\nu\rho\lambda\tau}^a(x) + O\left(\Theta^3\right). \tag{2.30}$$

the real deformed metric was introduced in [11] by the formula:

$$\hat{g}_{\mu\nu}(x,\Theta) = \frac{1}{2}\,\eta_{a\,b}\,\left(\hat{e}_\mu^a * \hat{e}_\nu^{b\,\dagger} + \hat{e}_\mu^b * \hat{e}_\nu^{a\dagger}\right). \tag{2.31}$$

2.3 Second order corrections to Reissner-Nordström de Sitter solution

Using the Ansatz (2.14)-(2.15), we can determine the deformed Reissner-Nordström de Sitter metric by the same method as the Schwarzschild metric was obtained in [11]. To this end, we have to obtain first the corresponding components of the tetrad fields $\hat{e}_\mu^a(x,\Theta)$ and their complex conjugated $\hat{e}_\mu^{a+}(x,\Theta)$ given by the Eqs. (2.27) and (2.30). With the definition (2.31) it is possible then to obtain the components of the deformed metric $\hat{g}_{\mu\nu}(x,\Theta)$.

Taking only space-space noncommutativity, $\Theta_{0i} = 0$ (due to the known problem with unitarity), we choose the coordinate system so that the parameters $\Theta^{\mu\nu}$ are given as:

$$\Theta^{\mu\nu} = \begin{pmatrix} 0 & \Theta & 0 & 0 \\ -\Theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mu,\nu = 1,2,3,0. \tag{2.32}$$

The non-zero components of the tetrad fields $\hat{e}^a_\mu(x,\Theta)$ are:

$$\begin{aligned} \hat{e}^1_1 &= \frac{1}{A} + \frac{A''}{8}\,\Theta^2 + O(\Theta^3), \\ \hat{e}^1_2 &= -\frac{i}{4}\left(A + 2\,r\,A'\right)\,\Theta + O(\Theta^3), \\ \hat{e}^2_2 &= r + \frac{1}{32}\left(7A\,A' + 12\,r\,A'^2 + 12\,r\,A\,A''\right)\Theta^2 + O(\Theta^3), \\ \hat{e}^3_3 &= r\sin\theta - \frac{i}{4}(\cos\theta)\,\Theta + \frac{1}{8}\left(2r\,A'^2 + rAA'' + 2AA' - \frac{A'}{A}\right)(\sin\theta)\,\Theta^2 + O(\Theta^3), \\ \hat{e}^0_0 &= A + \frac{1}{8}\left(2\,r\,A'^3 + 5\,r\,A\,A'\,A'' + r\,A^2\,A''' + 2\,A\,A'^2 + A^2\,A''\right)\Theta^2 + O(\Theta^3)\,, \end{aligned} \tag{2.33}$$

where A', A'', A''' are first, second and third derivatives of $A(r)$, respectively, with A^2 given in (2.19).

Then, using the definition (2.31), we obtain the following non-zero components of the deformed metric $\hat{g}_{\mu\nu}(x,\Theta)$ up to the second order:

$$\begin{aligned} \hat{g}_{1\,1}(x,\Theta) &= \frac{1}{A^2} + \frac{1}{4}\frac{A''}{A}\,\Theta^2 + O(\Theta^4), \\ \hat{g}_{22}(x,\Theta) &= r^2 + \frac{1}{16}\left(A^2 + 11\,r\,A\,A' + 16\,r^2\,A'^2 + 12\,r^2 A\,A''\right)\Theta^2 + O(\Theta^4), \\ \hat{g}_{33}(x,\Theta) &= r^2\,\sin^2\theta \\ &\quad + \frac{1}{16}\left[4\left(2\,r\,A\,A' - r\frac{A'}{A} + r^2\,A\,A'' + 2\,r^2\,A'^2\right)\sin^2\theta + \cos^2\theta\right]\Theta^2 + O(\Theta^4) \\ \hat{g}_{00}(x,\Theta) &= -A^2 - \frac{1}{4}\left(2\,r\,A\,A'^3 + r\,A^3\,A''' + A^3\,A'' + 2\,A^2\,A'^2 + 5\,r\,A^2\,A'\,A''\right)\Theta^2 + O(\Theta^4), \end{aligned} \tag{2.34}$$

For $\Theta \to 0$ we obtain the commutative Reissner-Nordström de Sitter solution with $A^2 = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2$.

We should mention that the expressions for the noncommutative corrections to the deformed tetrad fields and noncommutative metric elements are the same as the ones obtained in [11], although here we have also the electromagnetic field involved. The reason is that the first order noncommutative corrections to the electromagnetic field in the Seiberg-Witten map approach, i.e. $A^{(1)}_\mu$ is a pure gauge and thus can be gauged away [24]. As a result, in this order noncommutative corrections involving the electromagnetic field do not appear.

Now, if we insert A into (2.34), then we obtain the deformed Reissner-Nordström-de Sitter metric with corrections up to the second order in Θ. Its non-zero components are:

$$\begin{aligned}
\hat{g}_{11} &= \left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)^{-1} + \frac{(-2mr^3+3m^2r^2+3Q^2r^2-6mQ^2r+2Q^4)}{16r^2(r^2-2mr+Q^2)}\Theta^2\,, \quad (2.35)\\
\hat{g}_{22} &= r^2 + \frac{r^4-17mr^3+34m^2r^2+27Q^2r^2-75mQ^2r+30Q^4}{16r^2(r^2-2mr+Q^2)}\Theta^2\,,\\
\hat{g}_{33} &= r^2\sin^2\theta + \frac{\cos^2\theta(r^4+2mr^3-7Q^2r^2-4m^2r^2+16mQ^2r-8Q^4)}{16r^2(r^2-2mr+Q^2)}\Theta^2\\
&+ \frac{(-4mr^3+4m^2r^2+8Q^2r^2-16mQ^2r+8Q^4)}{16r^2(r^2-2mr+Q^2)}\Theta^2\,,\\
\hat{g}_{00} &= -\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right) + \frac{4mr^3-9Q^2r^2-11m^2r^2+30mQ^2r-14Q^4}{4r^6}\Theta^2\,.
\end{aligned}$$

2.4 Noncommutative scalar curvature and cosmological constant

It is well known that, in the commutative case, the scalar curvature of the vacuum solutions (like the Schwarzschild and Reissner-Nordström, when $\Lambda = 0$) vanishes, i.e. the corresponding space-time is Ricci-flat. It is interesting to study whether this property holds also in the noncommutative case. Moreover, this study is motivated also by the fact that in the commutative case the addition of a cosmological term leads to nonvanishing scalar curvatures even in the space devoid of any gravitational source. Should the scalar curvature not vanish for the deformed vacuum solutions, the noncommutative behaviour, in some sense or another, may naturally imitate the commutative solution with cosmological term.

The noncommutative Riemann tensor is expanded in powers of Θ as [14]:

$$\hat{F}^{ab}_{\mu\nu} = F^{ab}_{\mu\nu} + i\Theta^{\rho\tau}F^{ab}_{\mu\nu\rho\tau} + \Theta^{\rho\tau}\Theta^{\kappa\sigma}F^{ab}_{\mu\nu\rho\tau\kappa\sigma} + O(\Theta^3)\,, \tag{2.36}$$

where

$$F^{ab}_{\mu\nu\rho\tau} = \partial_\mu\omega^{ab}_{\nu\rho\tau} + (\omega^{ac}_{\mu}\omega^{db}_{\nu\rho\tau} + \omega^{ac}_{\mu\rho\tau} + \omega^{db}_{\nu} - \frac{1}{2}\partial_\rho\omega^{ac}_{\mu}\partial_\tau\omega^{db}_{\nu})\eta_{cd} - (\mu\leftrightarrow\nu) \tag{2.37}$$

and

$$F^{ab}_{\mu\nu\rho\tau\kappa\sigma} = \partial_\mu\omega^{ab}_{\nu\rho\tau\kappa\sigma} + (\omega^{ac}_{\mu}\omega^{db}_{\nu\rho\tau\kappa\sigma} + \omega^{ac}_{\mu\rho\tau\kappa\sigma} + \omega^{db}_{\nu} - \omega^{ac}_{\mu\rho\tau}\omega^{db}_{\nu\kappa\sigma} - \frac{1}{4}\partial_\rho\partial_\kappa\omega^{ac}_{\mu}\partial_\tau\partial_\sigma\omega^{db}_{\nu})\eta_{cd} - (\mu\leftrightarrow\nu)\,, \tag{2.38}$$

where $\omega^{ab}_{\mu\nu\rho}$ and $\omega^{ab}_{\mu\nu\rho\lambda\tau}$ are given by Eqs. (2.23) and (2.24), respectively, with $A = a$ and $B = b$. After we calculate $\hat{F}^{ab}_{\mu\nu}$ and have also $\hat{e}^a_\mu$ we can obtain the noncommutative scalar curvature:

$$\hat{F} = \hat{e}^\mu_a * \hat{F}^{ab}_{\mu\nu} * \hat{e}^\nu_b \tag{2.39}$$

where $\hat{e}^\mu_a$ is the inverse of $\hat{e}^a_\mu$ with respect to the star product, i.e. $\hat{e}^\mu_a * \hat{e}^b_\mu = \delta^b_a$. The general expression of the scalar curvature, expanded in powers of Θ, is:

$$\begin{aligned}
\hat{F} = F \,+\, &\Theta^{\rho\tau}\Theta^{\kappa\sigma}(e^\mu_a F^{ab}_{\mu\nu\rho\tau\kappa\sigma}e^\nu_b + e^\mu_{a\rho\tau\kappa\sigma}F^{ab}_{\mu\nu}e^\nu_b + e^\mu_a F^{ab}_{\mu\nu}e^\nu_{b\rho\tau\kappa\sigma} - e^\mu_{a\rho\tau}F^{ab}_{\mu\nu}e^\nu_{b\kappa\sigma}\\
&- e^\mu_{a\rho\tau}F^{ab}_{\mu\nu\kappa\sigma}e^\nu_b - e^\mu_a F^{ab}_{\mu\nu\rho\tau}e^\nu_{b\kappa\sigma}) + O(\Theta^4)\,.
\end{aligned} \tag{2.40}$$

Here, we have to calculate also $e^\mu_{a\rho\tau}$ and $e^\mu_{a\rho\tau\kappa\sigma}$, using:

$$\hat{e}^\mu_a = e^\mu_a - i\Theta^{\nu\rho} e^\mu_{a\nu\rho} + \Theta^{\nu\rho}\Theta^{\kappa\sigma} e^\mu_{a\nu\rho\kappa\sigma} + O(\Theta^3)\,. \tag{2.41}$$

The noncommutative scalar curvature for the Reissner-Nordström de Sitter solution is then obtained in the form:

$$\begin{aligned}\hat{F} &= 4\Lambda + \frac{96mr^5 - 552m^2r^4 - 72Q^2r^4 + 896m^3r^3 + 1174mQ^2r^3 - 2740m^2Q^2r^2}{32r^8(r^2-2mr+Q^2)}\Theta^2 \\ &+ \frac{-550Q^4r^{+}22362mQ^4r - 614Q^6}{32r^8(r^2-2mr+Q^2)}\Theta^2 \\ &+ \frac{(-16m^2r^4 + 28Q^2r^4 - 24mQ^2r^3 + 12Q^4r^2)}{32r^8(r^2-2mr+Q^2)}(\cot^2\theta)\ \Theta^2\end{aligned} \tag{2.42}$$

For the charge $Q = 0$ one obtains the scalar curvature for the deformed Schwarzschild de Sitter solution, which is also non-zero.

The very interesting feature of these scalar curvatures is that for finite values of the radius r they are non-zero for the pure Schwarzschild and Reissner-Nordström vacuum solutions (when $\Lambda = 0$), although asymptotically they do vanish. In a away, this situation locally mimics the presence of a nonvanishing cosmological term in the Einstein equations, where it is well known that the space-time curvature does not vanish even in the absence of any matter.

3. Noncommutativity corrections to the thermodynamical quantities of black holes

In this section we derive the corrections to the thermodynamical quantities due to the space-space noncommutativity.

For the Reissner-Nordstrom-de Sitter metric, there are four roots of $A^2(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2$, denoted r_i, with $i = 1, ..., 4$. In the Lorentzian section, $0 \le r < \infty$, the first root is negative and has no physical significance. The second root r_2 is the inner (Cauchy) black-hole horizon, $r = r_3 = r_+$ is the outer (Killing) horizon, and $r = r_4 = r_c$ is the cosmological (acceleration) horizon. The solution of

$$A^2(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2 = 0 \tag{3.1}$$

is found as a series expansion in the cosmological constant:

$$r = r_0 + a\Lambda + b\Lambda^2 + ...\,, \tag{3.2}$$

where r_0 is the Reissner-Nordström horizon radius because for $\Lambda = 0$ we obtain the Reissner-Nordström solution. We know that:

$$r_0 = m \pm \sqrt{m^2 - Q^2}\,. \tag{3.3}$$

Therefore, we obtain the following cosmological and black-hole horizon radius solutions with cosmological constant, respectively:

$$r_c = m + \sqrt{m^2 - Q^2} + \frac{(m + \sqrt{m^2 - Q^2})^5}{6(m(m + \sqrt{m^2 - Q^2}) - Q^2)}\Lambda\,, \tag{3.4}$$

$$r_+ = m - \sqrt{m^2 - Q^2} + \frac{(m + \sqrt{m^2 - Q^2})^5}{6(m(m + \sqrt{m^2 - Q^2}) - Q^2)}\Lambda\,. \tag{3.5}$$

On the other hand, for the Schwarzschild-de Sitter metric, with $A^2(r) = 1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2$, the number of positive roots (and thus the number of event horizons) depends on the ratio of m and $\frac{8\pi}{\Lambda}$. There are no event horizons if $m > \frac{1}{3\sqrt{\Lambda}}$. There is only one event horizon, $r_1 = \frac{1}{\sqrt{\Lambda}}$, if $m = \frac{1}{3\sqrt{\Lambda}}$. In the case $m < \frac{1}{3\sqrt{\Lambda}}$, there are two distinct event horizons:

$$r_2 = \frac{2}{\sqrt{\Lambda}}\cos\left(\frac{\pi}{3} + \frac{1}{3}\arctan\sqrt{\frac{1}{9m^2\Lambda} - 1}\right)\,, \tag{3.6}$$

$$r_3 = \frac{2}{\sqrt{\Lambda}}\cos\left(\frac{\pi}{3} - \frac{1}{3}\arctan\sqrt{\frac{1}{9m^2\Lambda} - 1}\right)\,. \tag{3.7}$$

It can be shown that $r_2 < r_1 < r_3$.

In the noncommutative case, we consider the corrected event horizon radius up to the second order as

$$\hat{r}_{1,2} = A_{1,2} + B_{1,2}\Theta + C_{1,2}\Theta^2\,. \tag{3.8}$$

Substituting the above $\hat{r}_{1,2}$ into the equation $\hat{g}_{00} = 0$, we obtain the corrected cosmological and black hole (Killing) event horizon radii respectively as solutions of this equation:

$$\begin{aligned}\hat{r}_1 &= m + \sqrt{m^2 - Q^2} + \frac{(m + \sqrt{m^2 - Q^2})^5}{6(m(m + \sqrt{m^2 - Q^2}) - Q^2)}\Lambda \\ &+ \frac{(6m^4 + \sqrt{m^2 - Q^2}(6m^3 - 8mQ^2) - 11Q^2m^2 + 5Q^4)}{8(8m^5 + \sqrt{m^2 - Q^2}(8m^4 - 8m^2Q^2 + Q^4) - 12m^3Q^2 + 4mQ^4)}\Theta^2\end{aligned} \tag{3.9}$$

and

$$\begin{aligned}\hat{r}_2 &= m - \sqrt{m^2 - Q^2} + \frac{(m + \sqrt{m^2 - Q^2})^5}{6(m(m + \sqrt{m^2 - Q^2}) - Q^2)}\Lambda \\ &+ \frac{(6m^4 - \sqrt{m^2 - Q^2}(6m^3 - 8mQ^2) - 11Q^2m^2 + 5Q^4)}{8(8m^5 - \sqrt{m^2 - Q^2}(8m^4 - 8m^2Q^2 + Q^4) - 12m^3Q^2 + 4mQ^4)}\Theta^2\end{aligned} \tag{3.10}$$

The distance between the corrected event horizon radii is given by following relation in an example case, when $m = 2Q$

$$\hat{d} = \hat{r}_1 - \hat{r}_2 = d - \Delta d = 2\sqrt{3}Q + \frac{51\sqrt{3}}{4Q}\Theta^2 \tag{3.11}$$

Therefore in the noncommtative space-time the distance between horizons is more than in the commutative case. Then we obtain from (3.11):

$$\frac{\Delta d}{d} = \frac{51\Theta^2}{8Q^2} \tag{3.12}$$

The ratio of this change due to the noncommutativity correction to the distance has a value which is much too small.

The modified Hawking-Bekenstein temperature and the horizon area of the Reissner-Nordström de Sitter black hole in noncommutative space-time to the second order in Θ are as follows, respectively:

$$\begin{aligned}
\hat{T}_+ = \frac{1}{4\pi}\frac{d\hat{g}_{00}(\hat{r}_1)}{dr} &= \frac{m^2 - m\sqrt{m^2-Q^2} - Q^2}{2\pi(m-\sqrt{m^2-Q^2})} \\
&+ \frac{Q^2(-4m^2Q^2\sqrt{m^2-Q^2} - 48m^5 + 68m^3Q^2 - 21mQ^4 + \sqrt{m^2-Q^2}Q^4)}{12\pi(m-\sqrt{m^2-Q^2})^4(-m^2 - m\sqrt{m^2-Q^2} + Q^2)}\Lambda \\
&+ (\frac{(448m^9 - 1648Q^2m^7 + 2112Q^4m^5 - 1091Q^6m^3 + 179Q^8m)}{16\pi(m+\sqrt{m^2-Q^2})^7[8m^5 - 12Q^2m^3 + 4Q^4m + \sqrt{m^2-Q^2}(8m^4 - 8Q^2m^2 + Q^4)]} \\
&+ \frac{\sqrt{m^2-Q^2}(-2240Q^2m^6 + 2597Q^4m^4 - 1053Q^6m^2 + 612m^8 + 84Q^8)}{16\pi(m+\sqrt{m^2-Q^2})^7[8m^5 - 12Q^2m^3 + 4Q^4m + \sqrt{m^2-Q^2}(8m^4 - 8Q^2m^2 + Q^4)]} \\
&+ \frac{(m^2-Q^2)^{3/2}(264Q^2m^4 - 473Q^4m^2 - 152m^6 + 51Q^6)}{16\pi(m+\sqrt{m^2-Q^2})^7[8m^5 - 12Q^2m^3 + 4Q^4m + \sqrt{m^2-Q^2}(8m^4 - 8Q^2m^2 + Q^4)]} \\
&+ \frac{(m^2-Q^2)^{5/2}(16Q^2m^2 - 12m^4)}{16\pi(m+\sqrt{m^2-Q^2})^7[8m^5 - 12Q^2m^3 + 4Q^4m + \sqrt{m^2-Q^2}(8m^4 - 8Q^2m^2 + Q^4)]})\Theta^2\,,
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
\hat{A}_+ = 4\pi\hat{r}_1^2 &= 4\pi((m-\sqrt{m^2-Q^2})^2 + \frac{(m-\sqrt{m^2-Q^2})(m+\sqrt{m^2-Q^2})^5}{3(m(m+\sqrt{m^2-Q^2}) - Q^2)}\Lambda) \\
&+ \frac{\pi(m+\sqrt{m^2-Q^2})[6m^4 - 11Q^2m^2 + 5Q^4 + \sqrt{m^2-Q^2}(6m^3 - 8Q^2m)]}{8m^5 - 12Q^2m^3 + 4Q^4m + \sqrt{m^2-Q^2}(8m^4 - 8Q^2m^2 + Q^4)}\Theta^2\,.
\end{aligned} \tag{3.14}$$

The corresponding quantities for the cosmological horizon are as follows, respectively:

$$\begin{aligned}
\hat{T}_c = \frac{-1}{4\pi}\frac{d\hat{g}_{00}(\hat{r}_2)}{dr} &= \frac{-m^2 - m\sqrt{m^2-Q^2} + Q^2}{2\pi(m+\sqrt{m^2-Q^2})} \\
&+ \frac{(-4m^2 - 4m)\sqrt{m^2-Q^2} + 5Q^2)(m+\sqrt{m^2-Q^2})}{12\pi(-m^2 - m\sqrt{m^2-Q^2} + Q^2)}\Lambda \\
&+ (\frac{(448m^9 - 1648Q^2m^7 + 2112Q^4m^5 - 1091Q^6m^3 + 179Q^8m)}{16\pi(m+\sqrt{m^2-Q^2})^7[8m^5 - 12Q^2m^3 + 4Q^4m + \sqrt{m^2-Q^2}(8m^4 - 8Q^2m^2 + Q^4)]} \\
&+ \frac{\sqrt{m^2-Q^2}(-2240Q^2m^6 + 2597Q^4m^4 - 1053Q^6m^2 + 612m^8 + 84Q^8)}{16\pi(m+\sqrt{m^2-Q^2})^7[8m^5 - 12Q^2m^3 + 4Q^4m + \sqrt{m^2-Q^2}(8m^4 - 8Q^2m^2 + Q^4)]} \\
&+ \frac{(m^2-Q^2)^{3/2}(264Q^2m^4 - 473Q^4m^2 - 152m^6 + 51Q^6)}{16\pi(m+\sqrt{m^2-Q^2})^7[8m^5 - 12Q^2m^3 + 4Q^4m + \sqrt{m^2-Q^2}(8m^4 - 8Q^2m^2 + Q^4)]} \\
&+ \frac{(m^2-Q^2)^{5/2}(16Q^2m^2 - 12m^4)}{16\pi(m+\sqrt{m^2-Q^2})^7[8m^5 - 12Q^2m^3 + 4Q^4m + \sqrt{m^2-Q^2}(8m^4 - 8Q^2m^2 + Q^4)]})\Theta^2\,,
\end{aligned} \tag{3.15}$$

$$\hat{A}_c = 4\pi\hat{r}_2^2 = 4\pi((m+\sqrt{m^2-Q^2})^2 + \frac{(m+\sqrt{m^2-Q^2})^6}{3(m(m+\sqrt{m^2-Q^2})-Q^2)}\Lambda) \\ + \frac{\pi(m+\sqrt{m^2-Q^2})[6m^4-11Q^2m^2+5Q^4+\sqrt{m^2-Q^2}(6m^3-8Q^2m)]}{8m^5-12Q^2m^3+4Q^4m+\sqrt{m^2-Q^2}(8m^4-8Q^2m^2+Q^4)}\Theta^2 . \tag{3.16}$$

According to the Bekenstein-Hawking formula the thermodynamic entropy of a black hole is proportional to the area of the event horizon $S = A/4$, where A is the area of the horizon. The corrected entropy due to noncommutativity for the black-hole horizon and the cosmological horizon are:

$$\hat{S}_+ = \frac{\hat{A}_+}{4} = \pi^2((m-\sqrt{m^2-Q^2})^2 + \frac{(m-\sqrt{m^2-Q^2})(m+\sqrt{m^2-Q^2})^5}{3(m(m+\sqrt{m^2-Q^2})-Q^2)}\Lambda) \\ + \frac{\pi\Theta^2(m+\sqrt{m^2-Q^2})[6m^4-11Q^2m^2+5Q^4+\sqrt{m^2-Q^2}(6m^3-8Q^2m)]}{8m^5-12Q^2m^3+4Q^4m+\sqrt{m^2-Q^2}(8m^4-8Q^2m^2+Q^4)} \\ + \frac{\pi\Theta^2(m+\sqrt{m^2-Q^2})[6m^4-11Q^2m^2+5Q^4+\sqrt{m^2-Q^2}(6m^3-8Q^2m)]}{4[8m^5-12Q^2m^3+4Q^4m+\sqrt{m^2-Q^2}(8m^4-8Q^2m^2+Q^4)]} \tag{3.17}$$

$$\hat{S}_c = \frac{\hat{A}_c}{4} = \pi^2((m+\sqrt{m^2-Q^2})^2 + \frac{(m+\sqrt{m^2-Q^2})^6}{3(m(m+\sqrt{m^2-Q^2})-Q^2)}\Lambda) \\ + \frac{\pi\Theta^2(m+\sqrt{m^2-Q^2})[6m^4-11Q^2m^2+5Q^4+\sqrt{m^2-Q^2}(6m^3-8Q^2m)]}{8m^5-12Q^2m^3+4Q^4m+\sqrt{m^2-Q^2}(8m^4-8Q^2m^2+Q^4)} \\ + \frac{\pi\Theta^2(m+\sqrt{m^2-Q^2})[6m^4-11Q^2m^2+5Q^4+\sqrt{m^2-Q^2}(6m^3-8Q^2m)]}{4[8m^5-12Q^2m^3+4Q^4m+\sqrt{m^2-Q^2}(8m^4-8Q^2m^2+Q^4)]} \tag{3.18}$$

If we consider the $Q = 0$ case, we obtain the corresponding quantities for the Schwarzschild de Sitter black holes.

4. Conclusions and discussions

Following Ref. [18], in the present paper we constructed a gauge theory for gravitation using the de Sitter group as the local symmetry. The gravitational field has been described by gauge potentials. The solutions of the gauge field equations were studied considering a spherically symmetric case. Assuming that the source of the gravitational field is a point-like mass electrically charged, we obtained the Reissner-Nordström solution. Then, a deformation of the gravitational field has been performed along the lines of Ref. [14] by gauging the noncommutative de Sitter $SO(4,1)$ group and using the Seiberg-Witten map. The corresponding space-time is also of Minkowski type, but endowed now with spherical noncommutative coordinates. We determined the deformed gauge fields up to the second order in the noncommutativity parameters $\Theta^{\mu\nu}$. The deformed gravitational gauge potentials (tetrad fields) $\hat{e}^a_\mu(x, \Theta)$ have been obtained by contracting the noncommutative

gauge group $SO(4,1)$ to the Poincaré (inhomogeneous Lorentz) group $ISO(3,1)$. Then, we have calculated these potentials for the case of the Reissner-Nordström solution and defined the corresponding deformed metric $\hat{g}_{\mu\nu}(x,\Theta)$. By finding the Reissner-Nordström solution, as well as the Schwarzschild solution in [11], for a noncommutative theory of gravity we came closer to plausible black-hole physics on noncommutative space-time. The event horizon of the black hole undergoes corrections from the noncommutativity of space as in Eq. (3.9). Since the noncommutativity parameter is small in comparison with the length scales of the system, one can consider the noncommutative effect as perturbations of the commutative counterpart. Then we have obtained the corrections to the temperature and entropy given in Eqs. (3.13) and (3.14).

The noncommutativity of space-time drastically changes the topology of the space-time in the vicinity of the source in the presence of gravitational fields, in the sense that the curvature is not zero, locally, while asymptotically is does vanish. This situation is, in a limited sense, similar to the effect of a nonvanishing cosmological term in usual Einstein's equations. It could not be a priori ruled out that in a fully consistent treatment of a noncommutative theory of gravity, without expansion in Θ, the effects of the cosmological constant could be less locally imitated by the noncommutativity. In any case, one can say that the NC corrections are of the same form as those arising from the quantum gravity effects [25].

The use of the Seiberg-Witten map for constructing the noncommutative gauge theory of gravity leads inevitably to some loss of information, at least concerning the "big picture", i.e. the global features of the space-time. The reason is that the compatibility of the commutative and noncommutative gauge transformation is required at algebraic level, for infinitesimal transformations. As a result, the noncommutative fields and, consequently, the observables, will always be expressed as power series in Θ, starting from the 0th order, which is inevitably the corresponding field or observable of the commutative theory. Thus, the Seiberg-Witten map approach is useful for calculating corrections, but some phenomena which may be peculiar to the entire noncommutative setting will be concealed. The phenomenon of UV/IR mixing [26] is a show-case for this. If we did perturbation in Θ, the nonplanar diagram (which is finite when using the whole star-product) would no more be finite and the planar diagram would remain UV-divergent.

This features of the Seiberg-Witten map may hide interesting aspects when it comes to singularities. In this paper we have obtained the same singularity structure for the Schwarzschild and Reissner-Nordström metrics: if the 0th order in Θ is singular, then higher order corrections could never cancel this singularity.[1]. This is valid for the deformed Ricci scalar curvature, as well as the Kretschmann invariant, $\hat{F}^{\mu\nu\rho\sigma}\hat{F}_{\mu\nu\rho\sigma}$, where $\hat{F}^{\mu\nu}_{\rho\sigma}$ is the deformed Riemann tensor. This is in sharp contradiction with the conclusions of Ref. [9], where a nonsingular de Sitter geometry was found in the origin. In Ref. [9] the noncommutativity is taken into account by one of its major effects, the infinite nonlocality which it produces - the source of gravitational field is not point-like, but it has a Gaussian extension, while the noncommutativity effects of the gravitational field have not been

[1]This phenomenon of disappearance of singularities in the noncommutative case reveals itself in the noncommutative instantons and solitons as nonperturbative solutions [27].

taken into account. A clear-cut conclusion can be provided only by a full treatment of the noncommutative theory of gravity.

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